

$$(f(x) \pm g(x))' = f'(x) \pm g'(x) \quad (C \cdot f(x))' = C \cdot f'(x)$$

$$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

$$(f(g(x)))' = f'(g) \cdot g'(x) \quad \left(\frac{df(x)}{dx}\right)_{x_0} = \left(\frac{1}{\frac{dy}{dx}}\right)_{y_0=f(x_0)}$$

$$(C)' = 0 \quad (x^a)' = a \cdot x^{a-1}$$

$$(\sin(x))' = \cos(x) \quad (\cos(x))' = -\sin(x)$$

$$(tg(x))' = \frac{1}{\cos^2(x)} \quad (ctg(x))' = -\frac{1}{\sin^2(x)}$$

$$(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}} \quad (\arccos(x))' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctg(x))' = \frac{1}{1+x^2} \quad (e^x)' = e^x \quad (\ln(x))' = \frac{1}{x}$$

$$(a^x)' = a^x \cdot \ln(a) \quad (\log_a(x))' = \frac{1}{x \ln(a)}$$

$$(sh(x))' = ch(x) \quad (ch(x))' = sh(x)$$

$$(arsh(x))' = \frac{1}{\sqrt{x^2+1}} \quad (arch(x))' = \frac{1}{\sqrt{x^2-1}}$$

$$\text{érintő: } y = f(x_0) + f'(x_0)(x - x_0)$$

$$\text{simulókör: } y = f(x) \quad y''(x_0) \neq 0 \rightarrow \rho(x_0) = \left| \frac{\sqrt{1+y'^2(x_0)^3}}{y''(x_0)} \right|$$

$$u = x_0 - \frac{1+y'^2(x_0)}{y''(x_0)} y'(x_0) \quad v = y(x_0) + \frac{1+y'^2(x_0)}{y''(x_0)}$$

$$x = x(t) \quad y = y(t) \quad t = t_0 \text{ helyen} \rightarrow \rho = \left| \frac{\sqrt{\dot{x}^2 + \dot{y}^2}}{\dot{x}\dot{y} - \dot{y}\dot{x}} \right|$$

$$u = x - \dot{y} \frac{\dot{x}^2 + \dot{y}^2}{\dot{x}\dot{y} - \dot{y}\dot{x}} \quad u = y + \dot{x} \frac{\dot{x}^2 + \dot{y}^2}{\dot{x}\dot{y} - \dot{y}\dot{x}}$$

$$\text{érintősík: } z = f(x_0, y_0) + f'_x(x_0, y_0)(x - x_0) + f'_y(x_0, y_0)(y - y_0)$$

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$$\int_g f(x, y, z) ds = \int_{t=a}^b f(x(t), y(t), z(t)) \sqrt{(\dot{x}(t))^2 + (\dot{y}(t))^2 + (\dot{z}(t))^2} dt$$

$$\int_g f(x, y, z) dx = \int_{t=a}^b f(x(t), y(t), z(t)) \dot{x}(t) dt$$

$$\sin(x) = \sum \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!} \quad (1+x)^a = \sum \binom{a}{n} x^n$$

$$\cos(x) = \sum \frac{(-1)^n \cdot x^{2n}}{(2n)!} \quad \binom{a}{n} = \frac{a \cdot (a-1) \cdot (a-2) \cdot \dots \cdot (a-n+1)}{n!}$$

$$\frac{1}{1-x} = \sum x^n \quad e^x = \sum \frac{x^n}{n!}$$

$$t = e^x \quad \text{helyettesítés} \quad \left[\begin{array}{l} t = tg \frac{x}{2} \quad \text{helyettesítés} \\ x = 2 \operatorname{arctg}(t) \quad dx = \frac{2dt}{1+t^2} \\ x = \ln(t), t > 0 \quad dx = \frac{dt}{t} \end{array} \right. \quad \left. \begin{array}{l} \sin(x) = \frac{2t}{1+t^2} \\ \cos(x) = \frac{1-t^2}{1+t^2} \end{array} \right.$$

$$\left[\begin{array}{ll} t = \arcsin(x) & \sqrt{1-x^2} = \cos(t) \quad x = \sin(t) \quad dx = \cos(t) dt \\ t = \operatorname{arsh}(x) & \sqrt{1+x^2} = \operatorname{ch}(t) \quad x = \operatorname{sh}(t) \quad dx = \operatorname{ch}(t) dt \\ t = \operatorname{arch}(x) \quad x \geq 0, \sqrt{x^2-1} = \operatorname{sh}(t) & x = \operatorname{ch}(t) \quad dx = \operatorname{sh}(t) dt \\ t = \operatorname{arch}(-x) \quad x \leq 0, \sqrt{x^2-1} = \operatorname{sh}(t) & x = -\operatorname{ch}(t) \quad dx = -\operatorname{sh}(t) dt \end{array} \right.$$

$$a_n = \frac{2}{p} \int_0^p f(x) \cos\left(\frac{2n\pi}{p}x\right) dx \quad b_n = \frac{2}{p} \int_0^p f(x) \sin\left(\frac{2n\pi}{p}x\right) dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{2n\pi}{p}x\right) + b_n \sin\left(\frac{2n\pi}{p}x\right) \right)$$

$$L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt \quad f(t-T) \rightarrow e^{-Ts} \cdot f(s)$$

$$e^{at} \rightarrow \frac{1}{s-a} \quad \sin(a \cdot t) \rightarrow \frac{a}{s^2+a^2} \quad e^{at} \cdot f(t) \rightarrow f(s-a)$$

$$\cos(a \cdot t) \rightarrow \frac{s}{s^2+a^2} \quad t^n \cdot f(t) \rightarrow (-1)^n \cdot \frac{d^n f(s)}{ds^n}$$

$$1(t) \rightarrow \frac{1}{s} \quad \operatorname{sh}(a \cdot t) \rightarrow \frac{a}{s^2-a^2} \quad (f(t))' \rightarrow s \cdot f(s) - f(0)$$

$$t^n \rightarrow \frac{n!}{s^{n+1}} \quad \operatorname{ch}(a \cdot t) \rightarrow \frac{s}{s^2-a^2} \quad (f(t))'' \rightarrow s^2 \cdot f(s) - s \cdot f'(0) - f''(0)$$

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$t = \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|}$ $\mathbf{b} = \frac{\dot{\mathbf{r}} \times \ddot{\mathbf{r}}}{|\dot{\mathbf{r}} \times \ddot{\mathbf{r}}|}$ $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$
 $\dot{\mathbf{r}}(t) = \dot{x}(t)\mathbf{i} + \dot{y}(t)\mathbf{j} + \dot{z}(t)\mathbf{k}$
 $\ddot{\mathbf{r}}(t) = \ddot{x}(t)\mathbf{i} + \ddot{y}(t)\mathbf{j} + \ddot{z}(t)\mathbf{k}$
 $\mathbf{n} = \mathbf{b} \times \mathbf{t}$

$s = \int_a^b |\dot{\mathbf{r}}(t)| dt = \int_a^b \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} dt$

$g = \frac{|\dot{\mathbf{r}} \times \ddot{\mathbf{r}}|}{|\dot{\mathbf{r}}|^3}$ $c = \frac{\dot{\mathbf{r}} \cdot \ddot{\mathbf{r}}}{|\dot{\mathbf{r}} \times \ddot{\mathbf{r}}|^2}$ $\rho = \frac{1}{g}$ $\mathbf{R} = \mathbf{r} + \frac{1}{g} \mathbf{n}$
(görcsület) (torzió) (simulókör sugár) g (simulókör közp.)

$$\mathbf{u}(\mathbf{r}) = u(x, y, z) \quad \nabla u = \mathbf{grad} u = \frac{\partial u}{\partial x} \mathbf{i} + \frac{\partial u}{\partial y} \mathbf{j} + \frac{\partial u}{\partial z} \mathbf{k}$$

$$\mathbf{v}(\mathbf{r}) = v_x(x, y, z)\mathbf{i} + v_y(x, y, z)\mathbf{j} + v_z(x, y, z)\mathbf{k}$$

áramvonalak: $\frac{\partial x}{v_x} = \frac{\partial y}{v_y} = \frac{\partial z}{v_z}$

$$(\mathbf{u} + \mathbf{v})' = \mathbf{u}' + \mathbf{v}'$$

$$(\mathbf{u} \cdot \mathbf{v})' = \mathbf{v} \circ \mathbf{grad} u + \mathbf{u} \cdot \mathbf{v}'$$

$$(\mathbf{u} \cdot \mathbf{v})' = \mathbf{v} \cdot \mathbf{u}' + \mathbf{u} \cdot \mathbf{v}'$$

$$(\mathbf{u} \times \mathbf{v})' = -\mathbf{v} \times \mathbf{u}' + \mathbf{u} \times \mathbf{v}'$$

$$\frac{d}{d\mathbf{r}} \mathbf{u}(\mathbf{v}(\mathbf{r})) = \frac{d\mathbf{u}}{d\mathbf{v}} \frac{d\mathbf{v}}{d\mathbf{r}}$$

$$\mathbf{v}' = \frac{d\mathbf{v}}{d\mathbf{r}} = \mathbf{D} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}$$

deriválttenzor

$$\text{div } \mathbf{v} = \nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\mathbf{rot} \mathbf{v} = \nabla \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

$$\text{div}(\mathbf{u} + \mathbf{v}) = \text{div } \mathbf{u} + \text{div } \mathbf{v}$$

$$\mathbf{rot}(\mathbf{u} + \mathbf{v}) = \mathbf{rot} \mathbf{u} + \mathbf{rot} \mathbf{v}$$

$$\text{div}(\mathbf{u} \cdot \mathbf{v}) = \mathbf{v} \cdot \mathbf{grad} u + \mathbf{u} \cdot \text{div } \mathbf{v}$$

$$\mathbf{rot}(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot \mathbf{rot} \mathbf{v} + \mathbf{v} \cdot \mathbf{grad} u$$

$$\text{div}(\mathbf{u} \times \mathbf{v})' = \mathbf{v} \cdot \mathbf{rot} \mathbf{u} - \mathbf{u} \cdot \mathbf{rot} \mathbf{v}$$

$$\mathbf{rot}(\mathbf{u} \times \mathbf{v})' = \mathbf{v}(\nabla \circ \mathbf{u}) - \mathbf{v} \cdot \text{div } \mathbf{v} + \mathbf{u} \cdot \text{div } \mathbf{v} - \mathbf{u}(\nabla \circ \mathbf{v})$$

$$\mathbf{rot} \mathbf{rot} \mathbf{v} = \mathbf{grad} \text{div } \mathbf{v} - \Delta \mathbf{v}$$

$$\mathbf{rot} \mathbf{grad} u = \mathbf{0}$$

$$\text{div } \mathbf{rot} \mathbf{v} = 0$$

$$\Delta = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

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$$\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$$

$$\int C \cdot f(x) dx = C \cdot \int f(x) dx$$

$$\int f'(x) \cdot g(x) dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x) dx$$

$$\int f(g(x)) \cdot g'(x) dx = F(g(x)), \text{ ha } F'(x) = f(x)$$

$$\int f(ax+b) dx = \frac{F(ax+b)}{a}, \text{ ha } F'(x) = f(x)$$

$$\int f'(x) \cdot f^a(x) dx = \frac{f^{a+1}(x)}{a+1}, a \neq -1$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)|$$

$$\int x^a dx = \frac{x^{a+1}}{a+1}, a \neq -1$$

$$\int \frac{1}{x} dx = \ln|x|$$

$$\int a^x dx = \frac{a^x}{\ln(a)}, \text{ ha } a > 0 \text{ és } a \neq 1$$

$$\int e^x dx = e^x$$

$$\int \sin(x) dx = -\cos(x)$$

$$\int \cos(x) dx = \sin(x)$$

$$\int \frac{1}{\cos^2(x)} dx = \text{tg}(x)$$

$$\int \frac{1}{\sin^2(x)} dx = -\text{ctg}(x)$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x)$$

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \arccos(x)$$

$$\int \frac{1}{1+x^2} dx = \text{arctg}(x)$$

$$\int \frac{-1}{1+x^2} dx = \text{arcctg}(x)$$

$$\int \text{ch}(x) dx = \text{sh}(x)$$

$$\int \text{sh}(x) dx = \text{ch}(x)$$

$$\int \frac{1}{\text{ch}^2(x)} dx = \text{th}(x)$$

$$\int \frac{1}{\text{sh}^2(x)} dx = -\text{cth}(x)$$

$$\int \frac{1}{\sqrt{x^2+1}} dx = \text{arsh}(x)$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \text{arch}(x)$$

$$\int \frac{1}{1-x^2} dx = \frac{\text{arth}(x) \text{ ha } |x| < 1}{\text{arcth}(x) \text{ ha } |x| > 1}$$

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